

Physics 110B
Homework # 9

- #1. The Jones vector for the
(a) elliptically polarized light
is given by:

$$\frac{1}{\sqrt{1+b^2}} \begin{pmatrix} 1 \\ be^{i\delta} \end{pmatrix}$$

The components can be written as follows:

$$E_x = \hat{x} \exp(i(\omega t - kx))$$

$$E_y = b \hat{y} \exp(i(\omega t - kx + \delta))$$

Or, looking at the real part of the above equations:

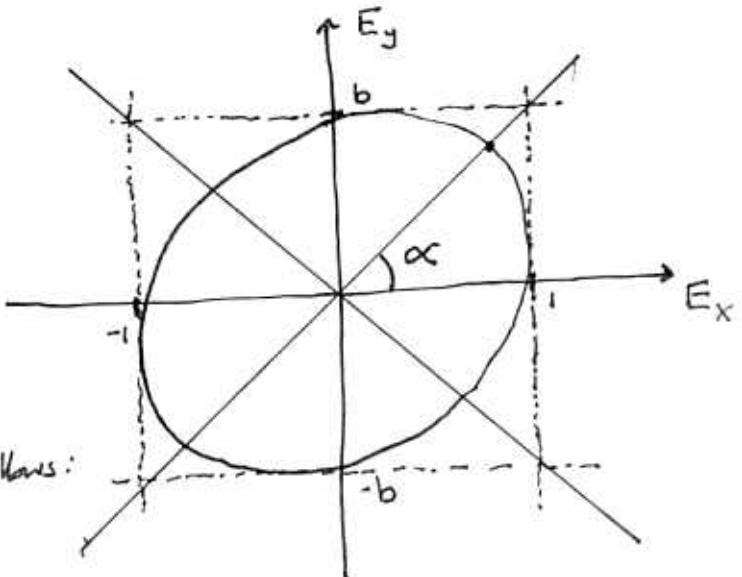
$$E_x = \cos(\omega t - kx), \quad E_y = b \cos(\omega t - kx + \delta)$$

$$\begin{aligned} \Rightarrow E_y &= b \cos(\omega t - kx) \cos \delta - b \sin(\omega t - kx) \sin \delta \\ &= b E_x \cos \delta - b \sin(\omega t - kx) \sin \delta \\ &= b E_x \cos \delta - b (1 - E_x^2)^{1/2} \sin \delta \end{aligned}$$

$$\Rightarrow \boxed{\frac{E_y^2}{b^2} + E_x^2 - \frac{2 E_y E_x \cos \delta}{b} = \sin^2 \delta}$$

Pedrotti
(Eq. 14-12)

The above equation describes an ellipse as given in the above figure.



$$\text{On the major axis: } \tan \alpha = \frac{E_y}{E_x}$$

The ellipse in the figure is contained in a rectangle with sides 2 and $2b$. The position of the major axis of the ellipse, at an angle α to the x-axis, is found as follows:

- The amplitude $E_x^2 + E_y^2$ is Max. on the major axis. Therefore at this point

$$\Delta(E_x^2 + E_y^2) = \frac{\partial f}{\partial E_y} dE_y + \frac{\partial f}{\partial E_x} dE_x = 0$$

$$① \Rightarrow E_y dE_y + E_x dE_x = 0$$

Now, similarly we maximize the equation for the ellipse above

$$\Delta\left(\frac{E_y^2}{b^2} + E_x^2 - \frac{2E_y E_x \cos\delta}{b}\right) = \frac{\partial g}{\partial E_x} dE_x + \frac{\partial g}{\partial E_y} dE_y = 0$$

$$② \Rightarrow \left(E_x - \frac{\cos\delta}{b} E_y\right) dE_x + \left(\frac{E_y}{b^2} - \frac{\cos\delta}{b} E_x\right) dE_y = 0$$

Combining eq① and eq② we have using the fact that $\tan\alpha = \frac{E_y}{E_x}$

$$1 - \frac{\cos\delta}{b} \tan\alpha = \frac{1}{b^2} - \frac{\cos\delta}{b} \cot\alpha$$

Since $\tan\alpha - \cot\alpha = \cot 2\alpha$, we find:

$$\boxed{\tan 2\alpha = \frac{2b \cos\delta}{1 - b^2}}$$

Pedrotti
(Eq 14-10)

(b) Using the results from Table 14-1 (p. 288) we see the sign of the imaginary term determines the handedness of the light:

$$\frac{1}{\sqrt{1+b^2}} \begin{pmatrix} 1 \\ be^{is} \end{pmatrix} = \frac{1}{\sqrt{1+b^2}} \begin{pmatrix} 1 \\ b\cos\delta + i b\sin\delta \end{pmatrix}$$

If $b\sin\delta > 0$ then it is left-handed

If $b\sin\delta < 0$ then the light is right-handed

(c) We break the Jones vector into linear and elliptical terms:

$$\frac{1}{\sqrt{1+b^2}} \begin{pmatrix} 1 \\ be^{is} \end{pmatrix} = \frac{1}{\sqrt{1+b^2}} \left(\begin{pmatrix} 1 - b\sin\delta \\ b\cos\delta \end{pmatrix} + \begin{pmatrix} b\sin\delta \\ i b\sin\delta \end{pmatrix} \right)$$

$$= \underbrace{\frac{1}{\sqrt{1+b^2}} \begin{pmatrix} 1 - b\sin\delta \\ b\cos\delta \end{pmatrix}}_{\text{linearly polarized}} + \underbrace{\frac{b\sin\delta}{\sqrt{1+b^2}} \begin{pmatrix} 1 \\ i \end{pmatrix}}_{\text{circularly polarized}}$$

Linearly polarized component is polarized in the direction:

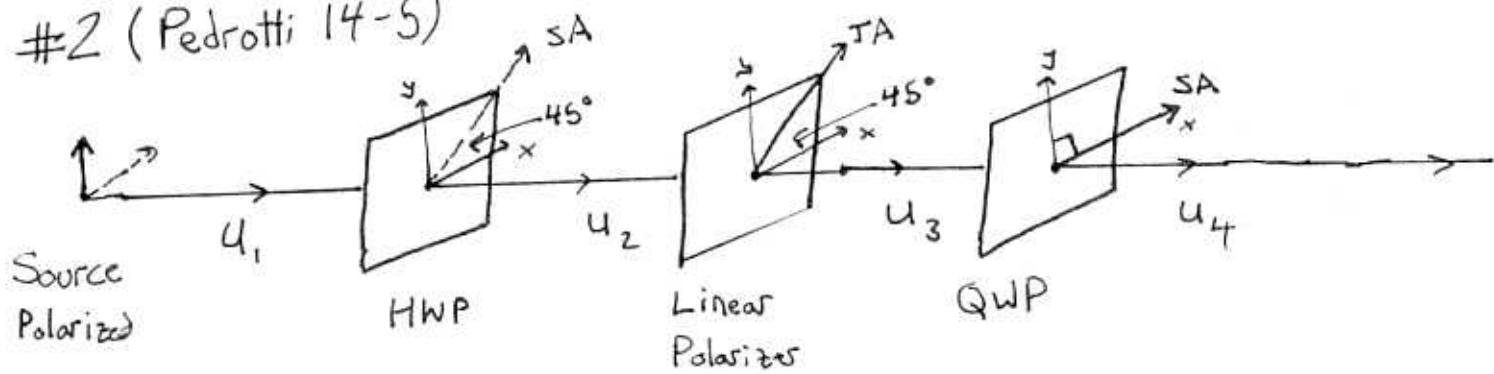
$$\tan \beta = \frac{b\cos\delta}{1 - b\sin\delta} \quad \Leftarrow \text{angle of polarization}$$

Hence,

$$\frac{\tan 2\alpha}{\tan \beta} = \frac{2 - 2b\sin\delta}{1 - b^2}$$

(4)

#2 (Pedrotti 14-5)



(a) The original light is represented by the following Jones vector:

$$u_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Before going through the HWP we rotate the light $-\pi/4$ and after coming out again we rotate the light $+\pi/4$:

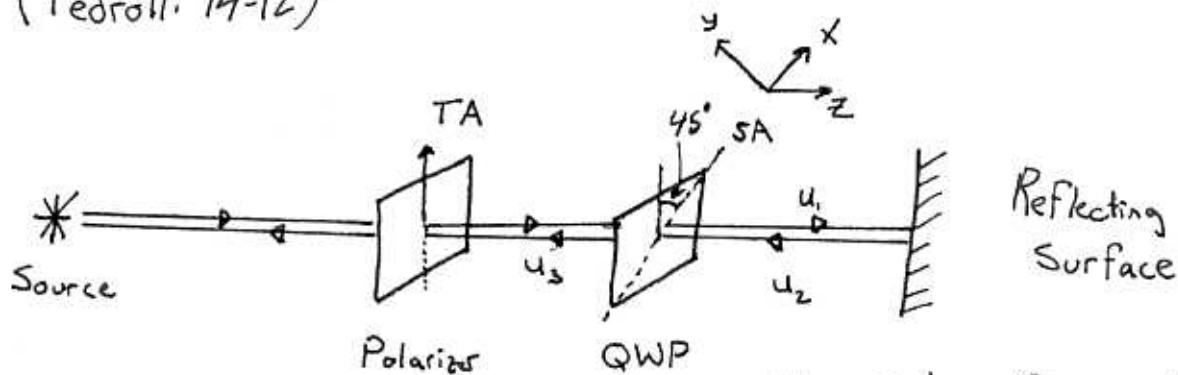
$$\begin{aligned} u_2 &= M_{\text{rotate } +\pi/4} M_{\text{HWP}} M_{\text{rotate } -\pi/4} u_1 \\ &= \begin{pmatrix} \cos \pi/4 & -\sin \pi/4 \\ \sin \pi/4 & \cos \pi/4 \end{pmatrix} \begin{pmatrix} e^{-i\pi/2} & 0 \\ 0 & -e^{-i\pi/2} \end{pmatrix} \begin{pmatrix} \cos(-\pi/4) & -\sin(-\pi/4) \\ \sin(-\pi/4) & \cos(-\pi/4) \end{pmatrix} u_1 \\ &= \frac{e^{-i\pi/2}}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} u_1 \\ &= \frac{e^{-i\pi/2}}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} u_1 = \frac{e^{-i\pi/2}}{2} \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} u_1 = \boxed{\frac{e^{-i\pi/2}}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = u_2} \end{aligned}$$

$$(b) u_3 = M_{\text{linear polarizer}} u_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} e^{-i\pi/2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \boxed{\frac{e^{-i\pi/2}}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = u_3}$$

$$(c) u_4 = M_{\text{QWP}} u_3 = e^{i\pi/4} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \frac{e^{-i\pi/2}}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \boxed{\frac{e^{-i\pi/4}}{2} \begin{pmatrix} 1 \\ -i \end{pmatrix} = u_4}$$

The final product is right circularly polarized light.

#3. (Pedrotti: 14-12)



Reflecting Surface

We make the slow axis of the QWP parallel to the x-axis.

u_1 = vector after; polarizer, QWP

u_2 = vector after; reflection

u_3 = vector after; reflection, QWP

This means the polarized light coming from the polarizer can be represented by the vector $E_{\text{pol}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

The QWP can be represented by the Jones matrix:

$$M_{\text{QWP}} = e^{i\pi/4} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

Thus, the light coming out of the QWP can be expressed as,

$$u_1 = M_{\text{QWP}} E_{\text{pol}} = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

So, we get right circularly polarized light

Upon reflection, the wave undergoes a 180° phase shift. However, both components of \bar{u}_1 undergo the same shift so it is of no consequence.

$$u_2 = u_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \quad \text{upon reflection still circularly polarized}$$

Now, the wave goes again through the QWP and we get,

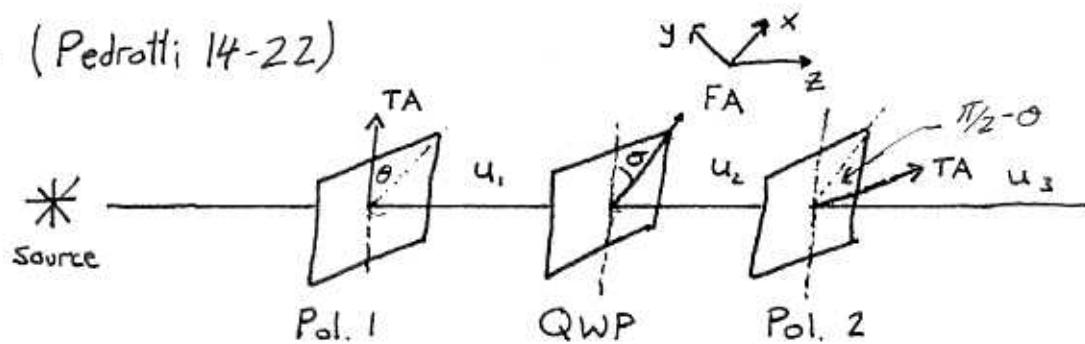
$$u_3 = M_{\text{QWP}} u_2 = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Lastly, the light goes back through the linear polarizer,

$$U_{\text{light out pol}} = M_{\text{pol.}} U_3 = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \boxed{0 = U_{\text{light out pol.}}}$$

Thus, we see that no light is reflected back to the source.

#4 (Pedrotti 14-22)



The x-axis has been chosen to be parallel to the fast axis of the QWP. Thus,

$$U_1 = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \Rightarrow U_2 = M_{\text{QWP}} U_1 = e^{i\pi/4} \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$

$$U_2 = e^{i\pi/4} \begin{pmatrix} \cos\alpha \\ i \sin\alpha \end{pmatrix}$$

Now going through the last polarizer using (Eq. 14-5) for $M_{\text{pol.2}}$,

$$U_3 = M_{\text{pol.2}} U_2 = e^{i\pi/4} \begin{pmatrix} \cos^2(\pi/2 - \omega) & \sin(\pi/2 - \omega)\cos(\pi/2 - \omega) \\ \sin(\pi/2 - \omega)\cos(\pi/2 - \omega) & \sin^2(\pi/2 - \omega) \end{pmatrix} U_2$$

$$= e^{i\pi/4} \begin{pmatrix} \sin^2\omega & \cos\omega\sin\omega \\ \cos\omega\sin\omega & \cos^2\omega \end{pmatrix} \begin{pmatrix} \cos\alpha \\ i\sin\alpha \end{pmatrix} = e^{i\pi/4} \begin{pmatrix} \cos\omega\sin^2\omega + i\cos\omega\sin^2\omega \\ \cos^2\omega\sin\omega - i\cos^2\omega\sin\omega \end{pmatrix}$$

$$\Rightarrow \boxed{U_{\text{emerging light}} = e^{i\pi/4} (1+i) \begin{pmatrix} \cos\omega\sin^2\omega \\ \cos^2\omega\sin\omega \end{pmatrix}}$$

The intensity of the emerging light equals,

$$I = U_3^T U_3 = e^{-i\pi/4} e^{i\pi/4} (1-i)(1+i) (\cos\alpha \sin^2\alpha, \cos^2\alpha \sin\alpha) \begin{pmatrix} \cos\alpha \sin^2\alpha \\ \cos^2\alpha \sin\alpha \end{pmatrix}$$

$$= 2 (\cos^2\alpha \sin^4\alpha + \cos^4\alpha \sin^2\alpha) = 2 \cos^2\alpha \sin^2\alpha$$

Thus, for initial intensity I_0 thru the above arrangement,

$I_{\text{emerging light}} = I_0 2 \cos^2\alpha \sin^2\alpha$

#5 The problem of N slits is done in Pedrotti see pages 341 - 345

From pedrotti we have the result:

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \left(\frac{\sin N\alpha}{\sin \alpha} \right)^2 \quad (\text{Eq } 16.32)$$

for very narrow slits $\beta \rightarrow 0$ thus $\frac{\sin \beta}{\beta} \Rightarrow \frac{\cos \beta}{\beta} = 1$

$$\textcircled{1} \quad I = I_0 \left(\frac{\sin N\alpha}{\sin \alpha} \right)^2 = I_0 \left(\frac{\sin 4\alpha}{\sin \alpha} \right)^2 \quad \text{For } N=4 \text{ slits}$$

$$\alpha = \frac{1}{2} k \alpha \sin \theta \approx \frac{1}{2} k a \theta \quad \begin{matrix} \theta \sim \text{being small} \\ a \sim \text{the slit separation} \end{matrix}$$

The maxima of the above equation are given by:

$$\alpha = \frac{p\pi}{N} \quad \text{or,} \quad a \sin \theta \frac{\pi}{\lambda} = \frac{p\pi}{N} \Rightarrow a \sin \theta = \frac{p\lambda}{N} \quad p=0, \pm 1, \pm 2, \dots$$

principal Maxima occur for $p=0, \pm N, \pm 2N, \dots$

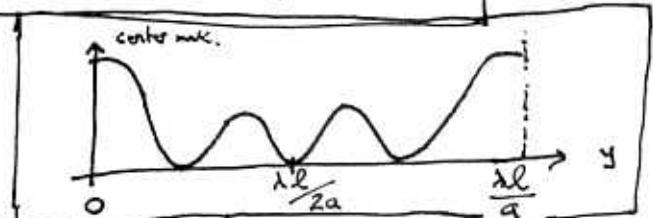
minima occur for = all other values

$$\Rightarrow \frac{p\lambda}{Na} = \frac{y}{l} \Rightarrow y = \frac{p\lambda l}{Na} \quad \begin{matrix} l \sim \text{distance from slit to screen} \\ y \sim \text{height of max. and min.} \end{matrix}$$

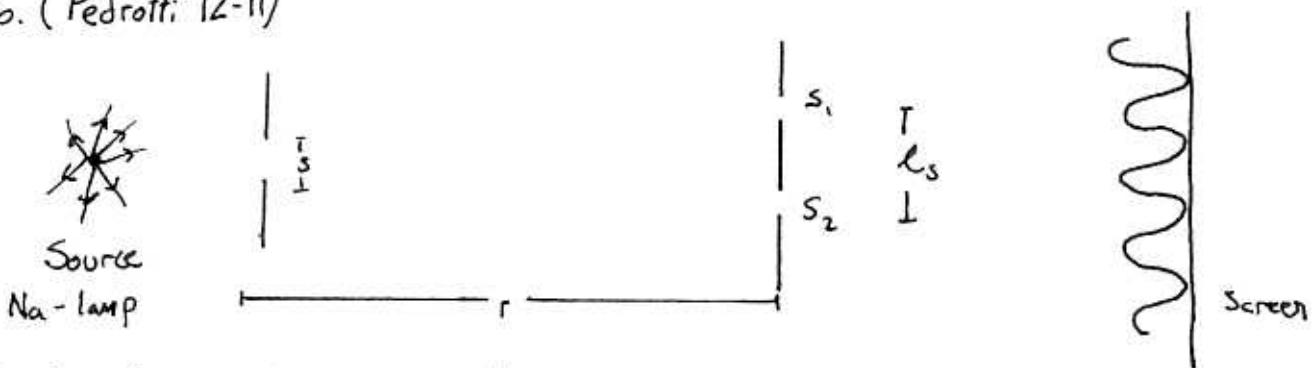
Thus, for the four slit Young's experiment we have:

$$y = \frac{p\lambda l}{4a} \quad \begin{matrix} \text{max occur for } p=0, \pm 4, \pm 8, \dots \\ \text{min occur for } = \text{all other integer values} \end{matrix}$$

Thus, we will get the following intensity pattern



#6. (Pedrotti: 12-11)



$$S = \text{diameter} = .5 \text{ MM} = 5 \times 10^{-4} \text{ m}$$

$$\lambda = 5890 \text{ \AA} = 5.89 \times 10^{-7} \text{ m}$$

$$r = 1 \text{ m}$$

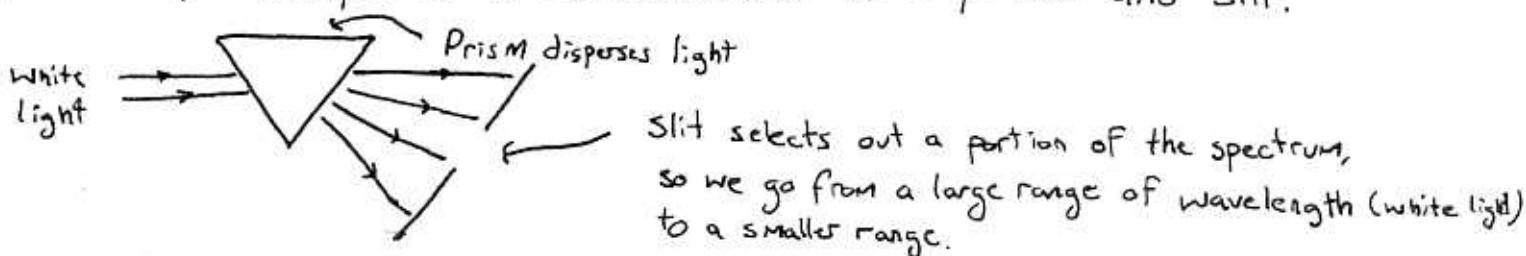
$$l_s^{\max} = ?$$

$$\Rightarrow l_s^{\max} = \frac{r\lambda}{s} = 1.178 \times 10^{-3} \text{ m} \quad (\text{Eq. 12-35})$$

$$l_s^{\max} = .118 \text{ cm}$$

#7. (Pedrotti: 12-13)

(a) A simple example of a monochromator is a prism and slit.



The linear dispersion is a measure of how well the monochromator disperses the input light. It tells one, for a given slit width, the range of wavelengths one gets out:

$$\text{Thus, } \lambda_0 = 500 \text{ nm}$$

$$w = 200 \mu\text{m} \quad \Rightarrow \quad \delta \lambda = \frac{\text{linear disp.}}{\text{slit width}} \cdot w = .4 \text{ MM}$$

$$\text{linear dispersion} = 2 \text{ nm/MM} \quad (\text{eq 12-17}) \quad \boxed{l_t = \frac{\lambda_0^2}{\delta \lambda} = 6.25 \times 10^{-4} \text{ M}}$$

$$\Rightarrow \boxed{\tau_0 = \frac{l_r}{c} = 2.08 \times 10^{-12} \text{ s}}$$

(b) $OPD = .4 \text{ mm}$

$$\Rightarrow \tau = \frac{OPD}{c} = 1.33 \times 10^{-12} \text{ s}$$

$$\Rightarrow |\gamma_{12}(\tau)| = 1 - \frac{\tau}{\tau_0} = \boxed{.361 = V} \quad (\text{Eq } 12-29)$$

(c) $I_{\max} = 100$

I_{\min} = Background Irradiance

$$V = .361 = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \Rightarrow I_{\min} = 47 \quad (\text{Eq } 12-32)$$

$$\Rightarrow \boxed{\Delta I = 53}$$

#8. (Pedrotti 12-14)

The height of the cylinder is determined by the temporal coherence.
The book gives the coherence length of white light (pg. 254):

$$h = l_r \approx 1000 \text{ nm} = 1.0 \times 10^{-6} \text{ m} = 1.8 \lambda_0 \quad (\lambda_0 = 550 \text{ nm})$$

The spatial coherence can be found as in the last example:

$$(\text{Eq } 12-36) \quad \ell_s = \frac{1.22 \lambda_0}{\theta} = \frac{1.22 \cdot (5.5 \times 10^{-7}) \text{ m}}{8.777 \times 10^{-3} \text{ rad}} = 7.69 \times 10^{-5} \text{ m}$$

$$\text{Diameter of base} = (2.5) \ell_s = 1.92 \times 10^{-5} \text{ m} \rightarrow r = 9.61 \times 10^{-6} \text{ m} \\ = 17.48 \lambda_0$$

$$\Rightarrow \boxed{h = 1.01 \times 10^{-6} \text{ m} \quad \text{or} \quad h = 1.83 \lambda_0} \\ A = \pi r^2 = 2.9 \times 10^{-10} \text{ m}^2 \quad d = 34.96 \lambda_0$$